Table 3 Parameter estimates from fits to simulated fBm plus trend (σ_H = 0.7, H = 0.4, Δt = 1.0 s, 128 points)

Case	a	σ_H	H
1) $a = 1.5$	1.486 ± 0.02	0.914 ± 0.04	0.301 ± 0.03
2) $a = 5.0$	4.986 ± 0.02	0.914 ± 0.04	0.301 ± 0.03

Table 4 Parameter estimates from fits to simulated fBm plus white noise (σ_H = 0.7, H = 0.4, Δt = 1.0 s)

Case	σ_m	σ_H	H
1) $\sigma_m = 0.025$ (128 pts)	0.271 ± 0.09	0.522 ± 0.09	0.599 ± 0.11
2) $\sigma_m = 0.25$ (128 pts)	0.353 ± 0.16	0.651 ± 0.17	0.461 ± 0.13
3) $\sigma_m = 0.25$ (200 pts)	0.402 ± 0.08	0.571 ± 0.10	0.556 ± 0.10
4) $\sigma_m = 1.00$ (128 pts)	——	——	——
5) $\sigma_m = 1.00$ (200 pts)	0.838 ± 0.19	0.899 ± 0.32	0.350 ± 0.13

The results of fitting to simulated fBm plus a trend and to simulated fBm plus white noise with the increment model are given in Tables 3 and 4. Case 4 in Table 4 would not converge until the number of data points was raised from $n = 128$ to 200 in case 5.

Fits to simulated fBm plus exponentially correlated noise with $\Delta t = 1.0$, $H = 0.4$, $\sigma_H = 0.7$, $c_1 = 0.5$, and $c_2 = 0.9$ failed with $n = 128$ and 200 measurements.

Fractional Brownian motion parameters were estimated from 128 points of 150-s averaged accelerometer data from Fig. 1:

$$H = 0.212 \pm 0.04, \qquad \sigma_H = 2.725 \pm 0.72 \tag{27}$$

which yields a log-log PSD slope of -1.424.

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Aircraft Controller Synthesis by Solving a Nonconvex Optimization Problem

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Introduction

A DESIGN technique for the synthesis of the pitch axis controller for an aircraft is presented in this Note. The proposed technique assumes a single-input multi-output system, and the type and order of the controller are assumed to be selected by the designer to reflect specific design requirements. The controller design problem is posed as a parameter optimization problem where the objective function is to minimize the infinity norm of a suitable error transfer function.

Formulation of the controller design problem as a parameter optimization problem with a time-domain performance criterion has been examined earlier,[1,2] and the problem has been solved using nonlinear programming (NLP) techniques. Unfortunately, the parameter optimization problem for fixed-order compensators is invariably nonconvex in the space of the design parameters, and hence, only a local minimum can be obtained. Direct design of low-order compensators using other techniques also leads to solutions having many local minima.[3]

In this Note, the controller design problem is formulated as a frequency-domain model-matching problem. The proposed technique is new both in the way the optimization problem is formulated and in the solution method used to find the optimal design parameters. In particular, we use the simulated annealing (SA) technique[4,5] to solve the optimization problem since this problem is nonconvex in the space of the design parameters. The application of the SA technique to solve controller design problems is new, and this Note explores the applicability of SA to controller design problems by considering a practical example from the field of aircraft control. For the sake of comparison, the solution to the optimization problem obtained using the NLP technique is also presented.

Problem Formulation

The specific problem to be considered here is the design of the pitch axis controller of an aircraft. It is assumed that the linearized longitudinal dynamics of the aircraft can be represented by the state-space description

$$\dot{x} = Ax + B\delta_e \qquad y = Cx + D\delta_e \tag{1}$$

where δ_e is the elevator input. The state-variable matrices for the design example considered are given in Table 1. Here it is assumed that the output variables available for feedback are pitch rate q, normal acceleration nz, and angle of attack α. For the purpose of illustrating the design technique, it is assumed that the controller structure to be implemented in the various loops have the form shown in Fig.1. In practice, the controller type and order can be completely arbitrary in each of the loops and, further, the number of feedback loops are also not fixed a priori. The only point to be kept in mind is that, given a single input, it is possible to arbitrarily shape only one output variable of interest.

Again for the purpose of illustration, it is assumed that the pitch rate is the output variable of interest, whose response has to be shaped to meet the handling quality specifications.[6] In particular, MIL-STD-1797A[6] requires that, for flight phases other than landing and take-off (which include air-to-air combat, rapid maneuvering,

Received Aug. 30, 1993, revision received Dec. 23, 1993; accepted for publication March 7, 1994.

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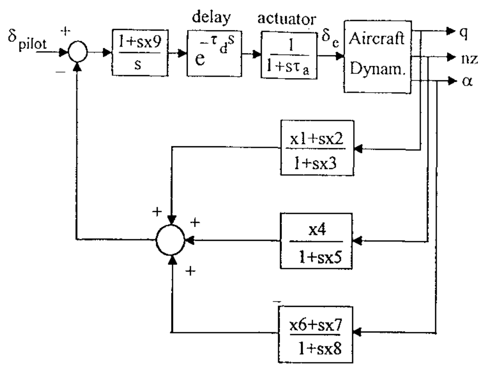

Fig. 1 Structure of pitch axis controller.

and in-flight refueling), the primary variable of interest is the short-period dynamics[7] of the pitch rate transfer function. In these flight phases, the closed-loop pitch rate dynamics is approximated by an equivalent second-order transfer function over the short-period frequency range of interest (0.1–10 rad/s) given by

$$\frac{q}{\delta_{\text{pilot}}} = \frac{\omega_n^2}{\theta} \frac{s+\theta}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} e^{-\tau s} \tag{2}$$

Henceforth, Eq. (2) will be referred to as the model transfer function $G_m(s)$.

For model matching, the objective function should minimize a suitable norm of the error transfer function between the actual closed-loop pitch rate dynamics and the short-period model over the desired frequency range of interest. Choosing the H_∞ norm of the error transfer function, the following objective function for the optimization problem is obtained:

$$\min_p \left\| W(s)\left[G_m(s) - G_q^p(s)\right] \right\|_\infty$$

$$= \min_p \sup_{\omega\in[\omega_l,\omega_u]} \left|G_m(j\omega) - G_q^p(j\omega)\right| \tag{3}$$

Here, $G_q^p(s) = q/\delta_{\text{pilot}}$ is the closed-loop pitch rate transfer function of the aircraft for the controller parameter vector $p = [x_1, x_2, \ldots, x_9]$ and $G_m(s)$ is the desired short-period model of the pitch rate transfer function given in Eq. (2). In our case a weighting transfer function $W(s)$ is a unity-gain ideal band pass filter in the short-period frequency range with its lower and upper cutoff frequencies being ω_l and ω_u, respectively. The minimization here is carried out over the space of the controller parameter vector p. It is to be observed that the objective function (3) can be interpreted as a weighted H_∞ norm minimization of the model-matching error under the assumption that $G_q^p(s)$ and $G_m(s)$ are stable transfer functions.

The additional specifications to be incorporated into the design procedure are the stability constraints given in terms of gain and phase margins. In the present study, phase and gain margins in the feedback loop of the response variable of interest were only considered. This is based on the classical one-loop-at-a-time analysis. Let us assume that the designer specifies desired gain and phase margins given by GM(d) and PM(d), respectively, in the q loop. Let the actual gain and phase margins obtained for a particular choice of controller parameter vector p be GM(a) and PM(a), respectively. Define

$$F_1 = \max\left[0, \frac{\text{GM}(d) - \text{GM}(a)}{\text{GM}(d)}\right] \tag{4a}$$

$$F_2 = \max\left[0, \frac{\text{PM}(d) - \text{PM}(a)}{\text{PM}(d)}\right] \tag{4b}$$

In general, since the gain and phase margin constraints for stability robustness are hard constraints in most controller design problems, the complete optimization problem that meets both the stability margins and the performance specifications can be formulated as

$$\mathcal{F} = \min_p \left[\sup_{\omega\in[\omega_l,\omega_u]} \left|G_m(j\omega) - G_q^p(j\omega)\right| + 10(F_1 + F_2)\right] \tag{5}$$

The constraints to be met in the above optimization problem are the range constraints on the controller parameters, which can be expressed as

$$p_i^{(l)} < p_i < p_i^{(u)}, \qquad i = 0, 1, \ldots, n_v \tag{6}$$

Here, n_v is the number of design variables and $p_i^{(l)}$ and $p_i^{(u)}$ are the lower and upper limits on these design variables. The complete optimization problem is then to minimize the objective function (5) subject to the constraint (6).

Optimization Package

The optimization problem formulated above is a constrained NLP problem that is nonconvex in the design parameter space. Hence, the use of gradient-based algorithms will only catch a local minimum. One way to avoid getting trapped in the local minimum is to use the SA algorithm, which has been effectively used to solve combinatorial optimization problems.[4] The extension of SA to optimization problems over continuous parameter space has also appeared in the literature,[5] and this extension has been used in the present study.

An optimization package called PITCHCAD has been developed by the author in the C language that solves the above optimization problem. For the sake of comparative study, both the SA algorithm[5] and an NLP algorithm using the inverse penalty function method have been implemented in the PITCHCAD program. The longitudinal dynamics of the aircraft have to be read from an input file, and the designer has to specify the form of the controller in each loop. Additionally, a pure time delay and an actuator model can also be included in the forward loop. The range constraints (6) for the design variables are read from the input file. The short-period model parameters of the transfer function given by Eq. (2) can be entered by the designer in an interactive session.

The optimization package developed incorporating the above design philosophy was used to synthesize the pitch axis controller for a relaxed statically stable aircraft. The state-variable matrices of the longitudinal dynamics of the aircraft for one flight condition are given in Table 1. For this flight condition, there is an unstable root at 0.1447. The parameters of the short-period model (2) were chosen as $\theta = 0.776$, $\zeta = 0.7$, $\omega_n = 3.0$, and $\tau = 0.08$ s, and the desired stability margins were chosen as GM(d) = 12 dB and PM(d) = 45 deg. The actuator time constant was chosen as $\tau_a = 0.05$ and the time delay in the forward loop as $\tau_d = 0.03$.

The optimization package was run on a PC-386/40 MHz with a math coprocessor. For the above problem, one run of the SA algorithm took approximately 12 min, whereas each run of the NLP algorithm took only 15 s. But the model matching obtained with the SA algorithm is significantly better than that obtained with the NLP algorithm. Several runs of the NLP algorithm with random starts did not produce results anywhere close to the SA solution. Identical conclusions were obtained by varying the number of feedback loops and the controller order in each loop. For the above problem, only one run of the SA algorithm was necessary and the following parameter vector was obtained:

$$p = [-0.5867, -0.2081, 0.4927, 0.1950, 0.3975, 0.1146,$$
$$0.02628, 0.2085, 0.6796]$$

Table 1 Aircraft longitudinal dynamics (0.5 m, 2.0 km, 1 g flight)

$$A = \begin{bmatrix} -0.7520 & 1.0 & 0.0 & -0.1540 \\ 0.7640 & -1.071 & 0.0 & 0.0223 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ -0.0451 & 0.0 & -0.0712 & -0.0254 \end{bmatrix}$$

$$B = [-0.3051 \quad -15.920 \quad 0.0 \quad -0.0512]^T$$

$$C = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ -2.1228 & 0.0735 & 0.0 & -0.3066 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$D = [0.0 \quad -0.0117 \quad 0.0]^T$$

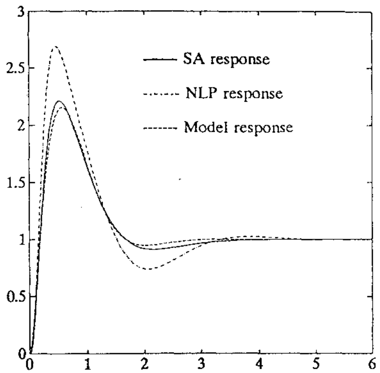

Fig. 2 Normalized pitch rate step responses.

For the above controller design parameters the gain margin is 20.5 dB and the phase margin is 51.2 deg in the q loop, and these meet the design requirements. The closed-loop pitch rate step response for the above controller parameters is shown in Fig. 2. It can be seen that it has excellent agreement with the step response of the model transfer function. Also shown in this figure is the pitch rate response obtained with the best NLP solution out of a total of 10 runs.

Conclusions

The direct synthesis of a feedback controller using parameter optimization techniques for the pitch axis of an aircraft has been described and illustrated with a design example in this Note. It has been shown that the nonconvex optimization problem solved using the SA algorithm produces significantly improved results in comparison with that using a constrained NLP algorithm, and the computational overheads are within reasonable limits.

Acknowledgment

The author thanks I. Hartmann, Institut für Regelungstechnik und Systemdynamik, Technische Universität Berlin, for help rendered in completion of this work.

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